

Gravitational lensing as a contaminant of the gravity wave signal in CMB

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Gravity waves (GW) in the early universe generate B -type polarization in the cosmic microwave background (CMB), which can be used as a direct way to measure the energy scale of inflation. Gravitational lensing contaminates the GW signal by converting the dominant E polarization into B polarization. By reconstructing the lensing potential from CMB itself one can decontaminate the B mode induced by lensing. We present results of numerical simulations of B mode delensing using quadratic and iterative maximum-likelihood lensing reconstruction methods as a function of detector noise and beam. In our simulations we find the quadratic method can reduce the lensing B noise power by up to a factor of 7, close to the no noise limit. In contrast, the iterative method shows significant improvements even at the lowest noise levels we tested. We demonstrate explicitly that with this method at least a factor of 40 noise power reduction in lensing induced B power is possible, suggesting that $r = P_h/P_R \sim 10^{-6}$ may be achievable in the absence of sky cuts, foregrounds, and instrumental systematics. While we do not find any fundamental lower limit due to lensing, we find that for high-sensitivity detectors residual lensing noise dominates over the detector noise.

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I. INTRODUCTION

CMB polarization is generated by Thomson scattering of photons off free electrons. To generate polarization one needs an anisotropic distribution of photons in the electron rest frame (more specifically, a non-vanishing quadrupole moment) and scattering that couples this angular anisotropy to the polarization (Thomson scattering in this case). These two conditions are satisfied during recombination, when most of the polarization signal is generated. After recombination the free streaming of photons leads to a large quadrupole moment, so if some fraction of the photons is rescattered when the universe is reionized then a new polarization contribution will be generated at the angular scale of horizon at reionization. One often refers to the two contributions as the recombination and reionization components, respectively.

Thomson scattering generates linear polarization only. This is usually expressed in terms of Stokes parameters Q and U , which are coordinate dependent. They can be decomposed into coordinate independent E and B type polarizations [1, 2, 3] with opposite parities. To linear order in perturbation theory, primordial scalar (density) perturbations can only generate E polarization, while gravitational waves (GWs) can generate both scalar E and pseudoscalar B . If the amplitude of the gravity waves is very small relative to scalars it cannot be isolated from the temperature anisotropies or E polarization due to cosmic variance. The B polarization is however insensitive to cosmic variance from scalar modes and is limited only by instrument noise, foregrounds and sky coverage. This fact generated attention as a potentially promising

tool to detect gravity waves and test inflation [4, 5]. The amplitude of gravity waves produced during inflation depends on its energy scale: higher energy scales give larger amplitude of gravity waves. In terms of the tensor to scalar power spectrum ratio one has $r = P_h/P_R \propto V_*^4$, where P_h is the tensor power spectrum, P_R is the scalar curvature power spectrum and V_*^4 is the energy density during inflation when the present Hubble scale exited the horizon. V_* has units of energy and has been termed the “energy scale” of inflation. We do not know this energy scale, but one of the possibilities is the scale of grand unification theories (GUTs) at $V_* \sim 10^{16}$ GeV. At this energy scale the gravity wave contribution is sufficiently large to be detectable. In this paper we define the tensor to scalar ratio in terms of their primordial power spectra, rather than the quadrupole moments as often defined. This has the advantage of relating the tensor to scalar ratio directly to the inflationary predictions independent of the cosmological parameters, which affect the CMB anisotropy spectra. For typical parameters we find $C_2^T/C_2^S \sim r/2$.

A future satellite mission dedicated to B type polarization has been identified as one of the NASA Einstein probes to be built over the next decade. One of the outstanding questions regarding such a mission is what are the required angular resolution and sensitivity to maximize the science output and at what level do systematics swamp the improvements in these. It has been pointed out [6] that gravitational lensing leads to a generation of B polarization even if none was present in the early universe. This could limit the extraction of gravity wave signal if unaccounted for [7, 8]. One can try to reconstruct the gravitational lensing potential using the non-Gaussian information present in the CMB data to improve the limits (the large-scale lensing B -modes exhibit higher-order correlations with small-scale polarization whereas inflationary GW B -modes do not). Recent work applied quadratic estimators [9] to argue that using

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these estimators leads to an order of magnitude improvement for the no noise experiment [10, 11]. This work has been interpreted as providing a fundamental limit to the gravity wave extraction due to the lensing. However, it is important to note that these papers do not rule out the possibility that better reconstruction methods may be constructed. Indeed, in a recent work we have shown that better estimators are indeed possible [12]. We have demonstrated explicitly that one can improve upon the no noise limit of the quadratic estimator. Indeed, in the idealized case of no noise, perfect resolution and lensing by a single scalar deflection potential, the lensing reconstruction can be achieved *exactly* and the lensing contamination can be removed completely. It is easy to see why this is so: lensing displaces photons, so one can write the final Q and U polarization in terms of the initial Q and U and lensing deflection angle \mathbf{d} . In the absence of gravity waves (null hypothesis) initial B vanishes and ignoring lensing rotation (i.e. taking $\nabla \times \mathbf{d} = \mathbf{0}$) the deflection angle can be written in terms of a scalar field with one degree of freedom at each point, $\mathbf{d} = \nabla \Phi$. In this case for N pixels there are $2N$ observables (two at each point, Q and U or E and B), and $2N$ unknowns, initial E and Φ . The number of unknowns thus equals the number of equations, so one can solve it exactly in the absence of noise. It is of course possible that there are degenerate modes that cannot be reconstructed; however, it has been shown that the fraction of modes that are degenerate is small and may even be zero. (See Appendix B of Ref. [12] for details.)

The idealized case discussed above is unrealistic, since in the real world noise always limits the achievable sensitivity (in Ref. [12] we found that the lensing rotation is not a limitation at the sensitivity levels that can be achieved in foreseeable future). At the same time, if the detector noise is large gravitational lensing is not limiting the GW detection anyways. As the detector noise is lowered our ability to clean the lensing contamination improves as well and if the scaling between the detector and residual lensing noise is linear then lensing may never be the dominating source of noise. The relevant question regarding the lensing contamination is thus not whether it provides a fundamental limit (which remains an open question), but rather how much does it degrade the gravity wave sensitivity for a given instrument noise and angular resolution. We will phrase this question in terms of a B -mode noise power spectrum: the minimum detectable tensor-to-scalar power spectrum ratio r that can be observed then scales linearly with it.

The lensing degradation issue is particularly interesting in the context of the required noise and angular resolution of a future CMB polarization satellite. There are important instrument and cost tradeoffs that need to be included in the design of such a mission. For example, since the bulk of the gravity wave signal is at large scales one could devise a high sensitivity low angular resolution instrument costing significantly less than the equivalent high angular resolution mission. However, in this case

one would not be able to reconstruct the lensing potential, making the lensing contamination more significant. The goal of this paper is to provide some guidance to these considerations, obtaining the lensing degradation factors as a function of detector noise and resolution. We will use both quadratic method [9] and the maximum-likelihood method [12], which improves upon quadratic in the low noise regime. We will ignore other sources of contamination such as foregrounds and instrument-related issues, which should be included in the full consideration of pros and cons of a mission design. Unless otherwise specified, we will use the fiducial cosmology of Ref. [12].

II. LENSING AND B -MODES

In this section, we briefly review techniques for lensing reconstruction from CMB polarization data. We then arrive at the main objective of this paper: to calculate, via simulations, the remaining “noise” level (including lensing residuals) after the lensing reconstruction has been performed and used to remove lensing B -modes. We compute $w_{P,eff}^{-1/2}$, the white noise power spectrum of the combined instrumental B -mode noise and lensing residuals that remains after lens cleaning.

A. Lensing reconstruction

While unique features of weak lensing effect on CMB power spectrum, such as smoothing of the peaks and transfer of power to small scales [13], can be used to deduce its presence, it is the nongaussian signatures that allow for the lensing reconstruction [14]. There are several lensing reconstruction methods proposed in the literature. Simple quadratic estimators [15] have been shown to be near optimal for the *Wilkinson Microwave Anisotropy Probe* (WMAP) and possibly *Planck* [16], but more efficient quadratic estimators have been shown to improve upon these for low noise, high angular resolution experiments [9, 17]. Recently we used likelihood techniques to develop an iterative estimator [12, 18], which further improves upon these if detector noise is sufficiently low, specially for the polarization data. The corresponding reconstruction errors have been computed as a function of noise and angular resolution in the absence of gravity wave contribution [12].

We use only polarization information (not temperature) for our lensing reconstruction in order to reduce the computation time required for the simulations. This is obviously a conservative assumption. But note that – although temperature anisotropies are potentially useful for measuring the convergence power spectrum, and for mapping the convergence on large angular scales ($l \lesssim 200$) – most of the lensing-induced B -modes come from smaller-scale convergence modes that are only accessible using polarization data [9]. Additionally, the

small-scale temperature anisotropies are contaminated by secondary processes such as the kinetic Sunyaev-Zeldovich/Ostriker-Vishniac effect and scattering during inhomogeneous reionization, which have the same spectral dependence as primary CMB fluctuations and hence can significantly degrade lensing reconstruction from temperature [19]. By comparison, CMB polarization is expected to be essentially free of secondary scattering contamination [19, 20].

Gravitational lensing re-maps the primary CMB polarization field according to:

$$\tilde{P}(\hat{\mathbf{n}}) = P(\hat{\mathbf{n}} - 2\nabla\nabla^{-2}\kappa(\hat{\mathbf{n}})), \quad (1)$$

where P is the primary polarization, $\tilde{P}(\hat{\mathbf{n}})$ is the lensed polarization in direction $\hat{\mathbf{n}}$, and κ is the lensing convergence field. The lensing has two effects on polarization that are of interest here. One is the generation of B -modes in the lensed polarization field; the other is the generation of an anisotropic two-point function:

$$\langle E_{\vec{l}_1} B_{\vec{l}_2} \rangle = \frac{1}{\pi^{1/2} l^2} C_{l_1}^{EE} \vec{l} \cdot \vec{l}_1 \sin(2\alpha) \kappa_{\vec{l}}, \quad (2)$$

where $\vec{l} = \vec{l}_1 + \vec{l}_2$, α is the angle between \vec{l}_1 and \vec{l}_2 , and higher-order terms in κ have been neglected. (The average is taken over CMB realizations only, not including noise.) The EB quadratic estimator developed by Ref. [9] is obtained by taking a minimum-variance linear combination of EB products subject to the constraint that the expectation value is $\kappa_{\vec{l}}$:

$$\hat{\kappa}_{\vec{l}}^{EB,(\text{quad})} = \frac{A_{EB}(l)}{2} \sum_{\vec{l}_1} \frac{C_{l_1}^{EE} E_{\vec{l}_1} B_{\vec{l}_2}}{(\tilde{C}_{l_1}^{EE} + N_{l_1}^{EE})(\tilde{C}_{l_2}^{BB} + N_{l_2}^{BB})}, \quad (3)$$

where A_{EB} is a normalization factor, $N_{l_1}^{EE}$ is the E -mode polarization noise power spectrum, and $\vec{l}_2 = \vec{l} - \vec{l}_1$. Ref. [9] gives similar quadratic estimators using the EE products; in this paper we use an $EE+EB$ quadratic estimator that is a minimum-variance weighting of the EE and EB products (although almost all of the information for low-noise experiments comes from EB). For the purpose of cleaning out the lensing B -mode, it is best to Wiener-filter the quadratic estimator [11] since this minimizes the mean squared difference between the estimated and “true” convergence maps:

$$\hat{\kappa}_{\vec{l}}^{WF} = \frac{C_{\vec{l}}^{\kappa\kappa}}{C_{\vec{l}}^{\kappa\kappa} + \sigma_{\vec{l}}^{\kappa\kappa}} \hat{\kappa}_{\vec{l}}^{(\text{quad})}, \quad (4)$$

where $\sigma_{\vec{l}}^{\kappa\kappa}$ is the power spectrum of the noise in the estimator $\hat{\kappa}_{\vec{l}}^{(\text{quad})}$. Lens cleaning consists of applying a de-lensing operation that inverts the mapping of Eq. (1).

The quadratic estimator using polarization has excellent performance, in particular it can ultimately recover the convergence map with signal-to-noise ratio exceeding unity out to $l \approx 1000$ if the instrument noise is sufficiently low. Nevertheless, as is shown in Ref. [9], the accuracy

of quadratic lens reconstruction is fundamentally limited by the B -mode cosmic variance. Put another way, the estimation of one convergence mode $\kappa_{\vec{l}}$ is contaminated by B -mode power generated by the other convergence modes $\kappa_{\vec{l}'}$ [21].

The iterative reconstruction algorithm [12] avoids this problem. While Ref. [12] introduces the iterative lensing reconstruction using the likelihood function, the algorithm used can also be thought of as follows: for low-noise experiments, the dominant source of uncertainty in the lens reconstruction is the above-mentioned cross-talk among different convergence modes. Once an estimate of the lensing field is available using Eq. (4), the CMB can be de-lensed using the estimated convergence field. Since the de-lensed CMB map has most of the lensing-induced B -mode removed, the cross-talk among the convergence modes is reduced; therefore, re-application of the quadratic estimator (with different weighting) results in an even lower-noise convergence map. The reader is referred to Ref. [12] for implementation details. The iterative procedure is only useful if the instrument noise is below the lensing-induced B signal (5.3 μK arcmin in the fiducial model at low l). However, as the instrument noise is reduced, the uncertainty in the iterative lensing reconstruction is also reduced; indeed, in the absence of foregrounds and the rotational component of the deflection field, it is not known whether there is *any* fundamental limitation to the reconstruction accuracy that can be obtained via the iterative estimator.

Since both the quadratic and iterative estimators assume absence of gravity waves one must be careful when applying them to the case with gravity waves. As discussed in the introduction, in the absence of noise one is simply solving for lensing assuming $B = 0$, which is of course violated if gravity waves are present. The most general approach is to solve for the tensor B modes, combined scalar and tensor E modes and lensing potential Φ simultaneously. However, since most of the lensing signal is at high l , while most of the intrinsic gravity wave signal is at low l , we can simplify and ignore the information from low l in the lensing reconstruction, but only use low l in B mode cleaning. We have tried cutoffs of $l = 50$ and $l = 150$ as well as no cutoff and found no significant difference between them. This is not surprising, since number of modes not used in the reconstruction is in both cases small compared to the total number of modes. Note that $l = 150$ corresponds roughly to the scale below which the gravity waves become negligible, while the reionization peak only contributes for $l \ll 50$.

While analytical (Fisher matrix) expressions for noise spectra exist for both quadratic and iterative methods, they may not be fully reliable in either case. The Fisher matrix tends to significantly underestimate the noise in the iterative method because the lensing reconstruction error is non-Gaussian and realization-dependent [12]. The analytical estimate for the quadratic estimator error estimate based on the approximation of Gaussian E and B modes [9] is more reliable, but still does not include

all terms in the covariance matrix. Some of these are analytically tractable and have been shown to increase the covariance by up to 20% [21, 22], but other terms (specifically higher order terms in the deflection angle \mathbf{d}) have not yet been computed. The safest approach is thus to use numerical Monte Carlo simulations, which by construction include all of the terms.

B. Simulations

We compute the post-cleaning B -mode noise power spectrum via simulation as follows. First a simulated pure E Gaussian CMB polarization field is generated using the unlensed C_l^{EE} power spectrum from CMBFAST [23] for the fiducial Λ CDM cosmology of Ref. [12]. Then a Gaussian convergence field κ is generated, and the polarization Q and U fields are re-mapped according to Eq. (1). We next add noise to the polarization field with power spectrum:

$$C_l^{EE}(\text{noise}) = C_l^{BB}(\text{noise}) = w_P^{-1} \exp \frac{l(l+1)\theta_{FWHM}^2}{8 \ln 2}, \quad (5)$$

where $w_P^{-1/2}$ is the instrument noise (typically measured in $\mu\text{K arcmin}$) and θ_{FWHM} is the full width at half maximum of the instrument's beam. The quadratic and iterative lens reconstruction algorithms are then applied as described in Ref. [12], with the modification that we remove $l < 150$ modes (technically we have set all rows and columns corresponding to $l < 150$ modes to zero in the σ -matrices of Ref. [12]). Once the estimated convergence field $\hat{\kappa}$ has been determined, it is used to “de-lens” the CMB polarization map, thereby yielding a map of the primary polarization field. We then compute the B -mode power by averaging $(B_1^{(\text{res})})^2$ over the $l < 150$ modes in the simulation. Each simulation is run on a 2048×2048 square grid with periodic boundary conditions and grid spacing of 1 arcmin, corresponding to a total area of 1165 square degrees. The residual B -mode power spectra quoted here are determined by averaging over 4 such simulations. We define the “effective noise” by:

$$w_{P,eff}^{-1} = C_l^{BB}(\text{residual}); \quad (6)$$

this has units of $(\mu\text{K arcmin})^2$ and represents the white noise from combined instrument noise and lensing residuals that limits detectability of the GW signal. Fig. 1 shows an input B polarization map assuming $C_2^T/C_2^S = 0.012$ (upper left), no cleaning map (upper right), quadratic cleaning map (lower left) and iterative cleaning map (lower right).

C. Results

The results for the error power spectrum of such a reconstruction are shown in Table I for a variety of noise

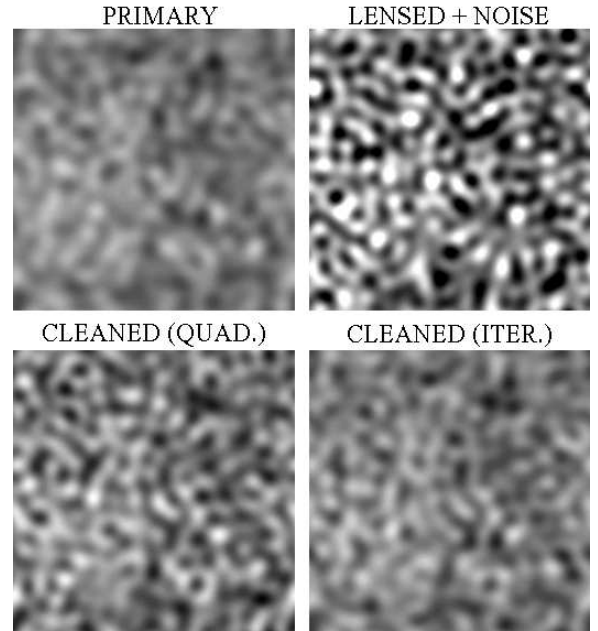


FIG. 1: Simulated extraction of a B mode from CMB data with noise $w_P^{-1/2} = 0.5\mu\text{K arcmin}$ and beam FWHM 4 arcmin. In each panel we have plotted the scalar quantity $B = \sum_l B_l e^{il \cdot \hat{n}}$. For clarity, only $l < 150$ modes are shown. The widths of the frames are 34 degrees, and the temperature scale runs from $-0.136\mu\text{K}$ to $+0.136\mu\text{K}$. *Upper left*: The primary B mode. *Upper right*: The B mode after lensing and addition of $0.5\mu\text{K arcmin}$ noise. *Lower left*: Recovered B mode after cleaning with the quadratic estimator. *Lower right*: Recovered B mode after cleaning with the iterative estimator.

and beam levels, for both quadratic and iterative methods. Since the spectra are close to white noise for $l < 150$, we only show the amplitudes and not the full spectrum. Note that the noise amplitude shown is the total noise and contains both lensing and detector noise contributions. Also note that for the 20' beam (and for larger beams), cleaning can actually make the B -mode noise worse because the de-lensing operation transfers power from high- l , unresolved CMB modes down to low l . In principle this problem can be circumvented by Wiener-filtering the CMB prior to the de-lensing operation; we have not implemented this because lens cleaning is not useful for such wide beams anyway.

The results can be divided into high, intermediate and low noise regimes. For high detector noise, $w_P^{-1/2} > 5\mu\text{K arcmin}$, lensing is a minor contributor to the total noise. Note that this noise level is still a factor of 100 (in power) lower than expected *Planck* polarization noise, so clearly any discussion of lensing induced noise in B is relevant only for a post-*Planck* CMB mission dedicated to polarization. For $w_P^{-1/2} = 6\mu\text{K arcmin}$ a 10' beam results in a total rms noise of $7.5\mu\text{K arcmin}$ and 7' beam in $7.3\mu\text{K arcmin}$ for either method. Without cleaning the combined lensing and detector noise

TABLE I: Residual B -mode contamination $w_{P,eff}^{-1/2}$ in $\mu\text{K arcmin}$ as a function of the instrument noise $w_P^{-1/2}$ and beam FWHM.

Beam FWHM	Instrument noise $w_P^{-1/2}$, $\mu\text{K arcmin}$					
	6.00	3.00	1.41	1.00	0.50	0.25
Quadratic estimator						
20'	8.73	7.13	6.70	6.48	5.71	4.75
15'	7.73	5.11	3.92	3.64	3.28	3.06
10'	7.49	4.79	3.53	3.22	2.88	2.68
7'	7.32	4.59	3.29	2.98	2.62	2.40
4'	7.20	4.39	3.02	2.69	2.30	2.09
2'	7.11	4.26	2.86	2.53	2.15	1.99
Iterative estimator						
7'	7.31	4.45	2.87	2.42	1.80	1.45
4'	7.17	4.23	2.56	2.07	1.39	1.00
2'	7.09	4.10	2.40	1.91	1.22	0.83

would be $8\mu\text{K arcmin}$, so cleaning hardly improves anything at all. Improvements appear when the detector noise drops below the lensing noise, which for our model is $5.3\mu\text{K arcmin}$. In the intermediate range ($2\text{--}5\mu\text{K arcmin}$) the quadratic estimator is very similar to the iterative method in terms of the residual noise. For example, for $w_P^{-1/2} = 3\mu\text{K arcmin}$ the residual noise is $4.5\mu\text{K arcmin}$ for $7'$ beam, a factor of 2 in power greater than the detector noise alone and a factor of 2 lower than no lens cleaning/large beam case. Going to a $4'$ beam marginally improves upon this.

Finally, in the low noise regime the iterative method clearly outperforms quadratic method. The quadratic method bottoms out roughly at $w_{P,eff}^{-1/2} = 2\mu\text{K arcmin}$, which is a factor of 7 improvement over the no-cleaning lensing noise of $w_{P,eff}^{-1/2} = 5.3\mu\text{K arcmin}$. This bottoming out of the quadratic method has lead to the suggestion that lensing noise limit may be fundamental and cannot be improved upon [10, 11]. However, this conclusion is only valid for the quadratic method, which is not the optimal method in the low detector noise regime. We find that the iterative method always reduces the overall noise as the detector noise is decreased, at least over the range tested with our simulations (which should cover the range of interest for the next generation CMB satellite dedicated to polarization). At the lowest noise and smallest beam ($0.25\mu\text{K arcmin}$, $2'$) tested in our simulation the lensing noise is reduced by more than a factor of 40. Further improvements are likely if the detector noise is reduced below $0.25\mu\text{K arcmin}$, but the iterative method becomes very computationally expensive and we have not explored these very low detector noise cases here.

For a given noise and lensing induced B -mode power spectrum the resulting uncertainty on r is:

$$\sigma_r^{-2} = f_{sky} w_{P,eff}^2 \sum_l \frac{2l+1}{2} \left(\frac{C_{Tl}^{BB}}{r} \right)^2, \quad (7)$$

where C_{Tl}^{BB} is the GW power spectrum of B modes (Fig. 2), and w_P is the inverse noise variance per solid

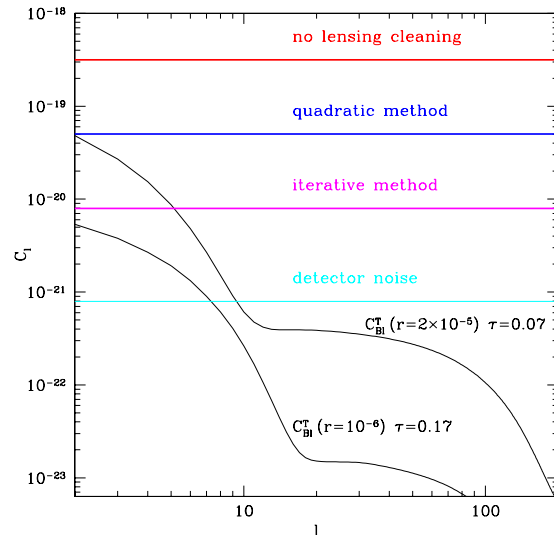


FIG. 2: Power spectra of noise for $2'$, $0.25\mu\text{K arcmin}$ instrument with no lensing cleaning, cleaning with quadratic method and cleaning with iterative maximum-likelihood method. Also shown are two theoretical power spectra for $r = 2 \times 10^{-5}$ and $r = 10^{-6}$. Assuming this instrument specifications and iterative method the former can be detected (at $2\text{-}\sigma$) both in reionization peak ($l < 20$) and in recombination peak ($l > 20$), while the latter is detectable for $l < 20$ only. The noise power spectra have been averaged over the $l < 150$ range.

angle per polarization. Since r is merely supplying the normalization of the tensor power spectrum, C_{Tl}^{BB}/r is fixed by the background cosmology. We are interested in large scales only, so the noise spectrum has been approximated as a constant and taken out of the sum. It is easy to see from this expression that the limit on $r = T/S$ is proportional to the noise weight per solid angle $w_{P,eff}^{-1}$. Therefore, the noise degradation factors are the same as r degradation factors. For partial-sky coverage, Eq. (7) must be modified to take into account sky cuts; while $\sigma_r \propto f_{sky}^{-1/2}$ for the recombination peak on degree scales, the reionization peak present at $l < 20$ exhibits a much more complicated dependence on the survey geometry due to cross-leakage of E and B modes induced by, e.g. the Galactic Plane cut [8]. Nevertheless, it remains true even in the presence of sky cuts that $\sigma_r \propto w_{P,eff}^{-1}$ if only the pure B -modes are used for GW searches. (But note that for the reionization peak, the uncertainty on r can be non-Gaussian due to the small number of modes, which complicates hypothesis tests for a GW contribution [8].)

III. DISCUSSION AND CONCLUSIONS

While the degradation factors can be computed independently of the theoretical spectrum, the actual achievable values of r depend on it. It is worth considering the

reionization and recombination peaks separately. While the reionization peak gives typically higher signal to noise if $f_{sky} = 1$, it depends sensitively on the Thomson scattering optical depth τ due to reionization, which is still rather uncertain (although this may improve with future observations of the reionization-induced TE correlation, which was recently detected by WMAP [24, 25]). In addition, incomplete sky coverage and foregrounds are particularly worrisome on large scales, so the reionization peak may be more difficult to observe than the recombination peak at $l \sim 100$.

As an example, for $l < 20$ and optical depth $\tau = 0.17$ one finds $\sigma_r = 5 \times 10^{-7}$ for $f_{sky} = 1$ and $w_{P,eff}^{-1/2} = 0.8\mu\text{K arcmin}$, which is the lowest noise level found in our simulations. The energy scale of inflation scales as $r^{1/4}$, so in this case the minimum energy scale one can detect at 3σ is $V_* \sim 10^{15}$ GeV. For full-sky coverage, the signal-to-noise scales somewhat less rapidly than τ^2 and reducing the optical depth to $\tau = 0.07$ increases σ_r by a factor of 3. Additional reduction will be caused by incomplete sky coverage, which will cause leakage of E into B [8, 26]; this degradation will be worst for models with late reionization because this pushes the B reionization peak to the low multipoles where sky-cut effects are most severe. For the recombination peak at $l > 20$ one has $\sigma_r = 10^{-5}$ for $w_{P,eff}^{-1/2} = 0.8\mu\text{K arcmin}$, which is a factor of 20 worse than reionization peak if $\tau = 0.17$ and a factor of 7 worse if $\tau = 0.07$. The corresponding energy scale that can be detected at 3σ is $V_* \sim 2.3 \times 10^{15}$ GeV.

To summarize, in this paper we present a detailed nu-

merical study of how well can one clean B -type polarization of the contamination caused by gravitational lensing. We find that quadratic methods are able to reduce the noise power by up to a factor of 7, while our iterative method is able to reduce the noise significantly beyond that, at least a factor of 40 in our simulations. With this method we do not find any fundamental lower limit caused by gravitational lensing, in the sense that over the range of parameters considered, reducing the instrument noise always leads to a reduction in lensing noise as well. However, the scaling is sublinear, so at low detector noise levels lensing noise dominates over instrument noise. With the noise cleaning one can achieve tensor to scalar ratios as low as $r \sim 10^{-6}$ and possibly even lower, which should allow us to differentiate between different models of structure formation with high precision. In particular, inflationary models with an energy scale significantly below 10^{15} GeV, as well as cyclic/ekpyrotic models [27, 28, 29] predict that the gravity waves should be negligible and so their predictions could be falsifiable with the future CMB polarization studies.

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